FORMATION OF FIBERS FROM TUBULAR SEMIFINISHED PRODUCTS

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Steady-state regimes of fiber formation from a tubular semifinished product are numerically modeled. The dependence of the form of the stream in the deformation zone on process parameters is obtained.

The need to study mathematical models of fiber formation stems from the requirements of modern technologies used for the production of chemical and optical fibers.

Such methods of fiber formation as drawing from a solid semiproduct or through a die are widely used. However, due to the expanding use of fluoride glasses, the method of producing fibers from a tubular semifinished product [1] is receiving increasing attention. Depending on the parameters of the process, this method is characterized by the following cases: absence of collapse of the capillary; collapse; irregular collapse. The latter is due to the nonsteadiness of the process, which results in the formation of microscopic pores in the center of the fiber. These pores are located along the fiber axis. The present article is devoted to study of the effect of instability on collapse of the fiber material.

We will examine steady-state regimes of fiber formation from tubular semifinished products and establish the main parameters of the process that affect collapse of the capillary. We distinguish two regions in the deformation zone. In the first region (from the beginning of deformation to the point of collapse), the fiber is still a capillary. In the second region (from the point of collapse to the end of the deformation zone), it is already solid. The viscosity of the semifinished product (SP) and the finished microcapillary or fiber is assumed to be infinitely large and is a known function of temperature. The temperature distribution is given. The liquid is isotropic and its motion is assumed to be axisymmetric. Considering that heat transfer to the SP inside the heater and subsequent cooling of the fiber occur by radiation and that the mean free path of the radiation exceeds the transverse dimension of the stream, we can assume the temperature of the liquid to be constant at all points of the cross section of the stream. This means that the distribution of temperature and, thus, viscosity depend only on the longitudinal coordinate and is described by an assigned function $\eta(z)$. It is clear from the formulation of the problem that $\eta(z)$ is a smooth function which approaches infinity as $z \to \pm\infty$.

The main approximations with which the mathematical model of the process is formulated are the assumptions of a prescribed temperature distribution and the smallness of the angle of inclination of the boundaries of the stream (transition from semifinished product to finished fiber).

Here, we determine the dependence of the form of the stream in the deformation zone on the parameters of the process.

We proceeded on the basis of the mathematical model proposed in [2]. The process is described by the Navier—Stokes equations and continuity equation. We assigned the thickness of the wall h_0 (Fig. 1), the mean radius of the tube $\bar{r}_0$, the feed of the tube u_0, and the rate of extraction of the finished product u_∞. The solution takes into account surface tension σ and the pressure gradient $\Delta p = p_1 - p_2$ between the channel and the environment; both quantities are assumed to be independent of the longitudinal coordinate z.

In light of the above, the dynamics of the process in the first region, i.e. from the beginning of deformation to the point of collapse, is described by the following system of equations with boundary conditions:

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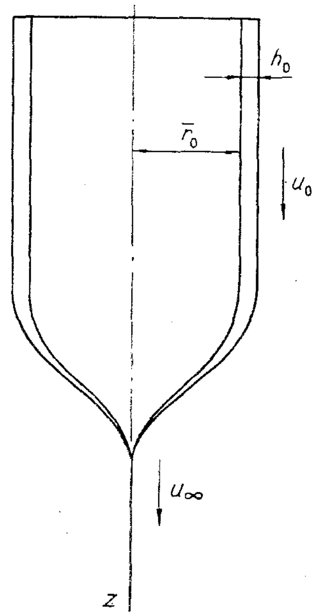


Fig. 1. Sketch of the stream in the deformation zone.

$$\eta du/dz - \alpha u = -\sigma/3h; \tag{1}$$

$$2udh/dz + hdu/dz = \sigma/\eta - (\bar{r}\Delta p/2\eta)[1 - (h^2/4\bar{r}^2)]; \tag{2}$$

$$\bar{r}hu = \beta; \tag{3}$$

$$u|_{z=-\infty} = u_0;\ \bar{r}|_{z=-\infty} = \bar{r}_0;\ h|_{z=-\infty} = h_0, \tag{4}$$

where α and β are unknown constants. We find from Eq. (3) with $z = -\infty$ and from (4) that $\beta = \bar{r}_0 h_0 u_0$. Given the conditions of our problem, we cannot assume that the fiber has thin walls. This would allow us to ignore the quantity $(h/\bar{r})^2$ in (2) relative to unity (as was done in [2]), since the wall thickness h and the mean radius $\bar{r}$ are of the same order of magnitude near the point of collapse of the microcapillary. Thus, below we will solve the problem in the complete formulation.

After changing over to dimensionless variables:

$$Z^* = z/l;\ H^* = h/\sqrt{h_0\bar{r}_0};\ R^* = \bar{r}/\sqrt{h_0\bar{r}_0};\ U^* = u/u_0;\ \mu^* = \eta_0/\eta, \tag{5}$$

where the effective length of the heating zone ℓ is determined by the equality

$$l/\eta_0 = \int_{-\infty}^{+\infty} dz/\eta(z), \tag{6}$$

the following dimensionless parameters enter into the equations and boundary conditions

$$U^*_\infty = u_\infty/u_0;\ W = \ln U^*_\infty;\quad \gamma = \alpha l/\eta_0;$$
$$Q = \sigma l/\eta_0 u_0\sqrt{h_0\bar{r}_0};\quad P = \Delta pl/2\eta_0 u_0. \tag{7}$$

In the dimensionless variables (with the superscript * omitted), Eqs. (1-3) and boundary conditions (4) are written in the form

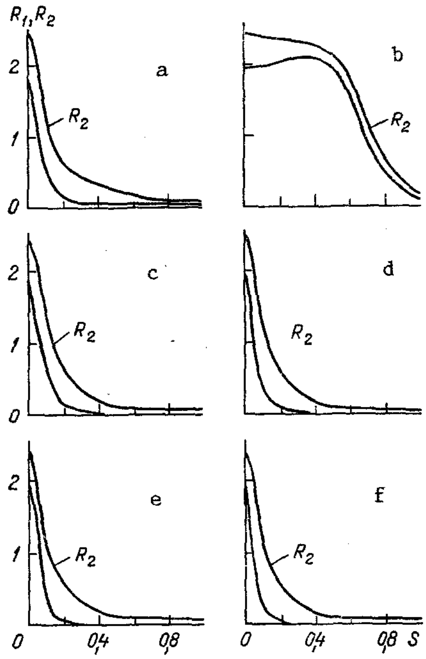
$$(1/R)\, dR/dS = -(\gamma/2 + Q/3 - PR^2[1 - 1/4R^4U^2]/2); \tag{8}$$

$$(1/U)\, dU/dS = (\gamma - QR/3); \tag{9}$$

$$HUR = 1; \tag{10}$$

$$H|_{S=0} = \sqrt{K};\quad R|_{S=0} = \sqrt{(1/K)};\ U|_{S=0} = 1, \tag{11}$$

where Q is the ratio of surface tension to the viscous forces; the quantity P characterizes the ratio of the pressure gradient between the channel and the environment to the viscous forces;



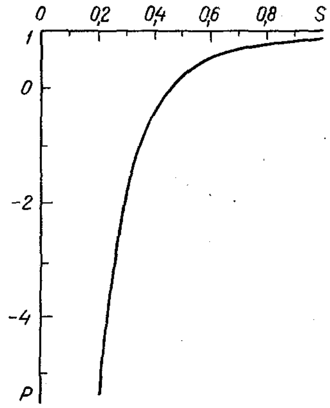


Fig. 2 Fig. 3

Fig. 2. Forms of hollow and solid streams with different process parameters: a) Q = 10.06231; γ = 12.35771; K = 0.2; P = 1; W = 11.51293; b) Q = 10.06231; γ = 9.802176; K = 0.2; P = 5; W = 4.60517; c) Q = 10.06231; γ = 12.21116; K = 0.2; P = 0; W = 11.51293; d) Q = 10.06231; γ = 12.10252; K = 0.2; P = −1; W = 11.51293; e) Q = 10.06231; γ = 11.99141; K = 0.2; P = −2; W = 11.51293; f) Q = 10.06231; γ = 11.90251; K = 0.2; P = −3; W = 11.51293.

Fig. 3. Dependence of the dimensionless coordinate of the point of collapse on the pressure parameter P for Q = 10.06231; K = 0.2; W = 11.51293.

$$S = \int_{-\infty}^{\xi} \mu(\xi)\, d\xi. \tag{12}$$

Here, the range (−∞; +∞) of Z corresponds to the range (0, 1) of S; μ(ξ) is the dimensionless distribution of the yielding of the material; K is the ratio of the thickness of the wall of the tube to its mean radius, referred to as the capillarity coefficient of the semi-finished product.

Thus, in the first region − where S changes from S = 0 to the capillary collapse point S_{col}, the process is described by system (8-10) with boundary conditions (11). In the second region − where the fiber is already solid, i.e. from $S = S_{col}$ to S = 1 − the mean radius is equal to

$$R(S) = H_i(S)/2; \quad S_{col} \leqslant S \leqslant 1. \tag{13}$$

Also, since the fiber is already solid, the pressure gradient between the channel and the environment is equal to zero. Thus, the dimensionless parameter P should also be equal to zero.

For the second region, the initial system of equations for the mean radius R and the longitudinal coordinate U appear as follows in dimensionless form:

$$(1/R)\, dR/dS = -(\gamma/2 + QR/3); \tag{14}$$

$$R^2 U = 1/2. \tag{15}$$

The boundary condition for system (14-15) is imposed at the point $S = S_{col}$ and is found from the solution of the problem in the first region.

To determine the parameter γ, we use the auxiliary condition $U|_{S=1} = \exp W$, where W is the natural logarithm of the ratio of the rate of fiber extraction to the feed of the semifinished product.

The problem was solved numerically. The parameter γ was found by the method of subdivision of the interval into polynomials. In each iteration, i.e. with a fixed value of γ, system (8-10) with boundary conditions (11) was solving by the Hemming method. We found the coordinate of the collapse point (if it existed) from condition (13). The values of mean radius and the longitudinal velocity at the collapse point then served as boundary conditions for system (14-15), and we used them to find the mean radius R and the longitudinal velocity U of the solid fiber from the collapse point to the end of the deformation zone (S = 1). This determination was made in dimensionless form.

The results of the calculations show that there are two characteristic regimes of fiber formation. Capillary collapse occurs in the first regime but not in the second. Typical profiles of the stream in the pulling region are shown in Fig. 2a and b, for the case when the fiber does not collapse completely and in Fig. 2c-f for the case when it does.

It can be seen that with fixed values for the capillarity coefficient of the semifinished product K, the surface tension parameter Q, and the fiber extraction parameter W, the process is significantly influenced by the parameter P characterizing the pressure gradient between the channel and the environment. Figure 3 shows the dependence of the dimensionless coordinate of the collapse point on pressure P. It is evident that a small change in the dimensionless pressure parameter P from 0 to 1 is accompanied by a substantial change in the coordinate of the collapse point – from $S_{col} = 0.46$ to the end of the deformation zone S = 1. The capillary does not collapse at P = 1. It is significant that variation of the pressure parameter P from –5 to 0 also does not greatly affect the coordinate of the collapse point. Here, the point is located in the upper half of the pulling region, i.e. above the point of the temperature maximum.

The case in which collapse does not occur is interesting. It is shown in Fig. 2b. With a low rate of fiber extraction W = ln (100) and P = 5, we find that the capillarity coefficient K of the semifinished product changes from 0.2 to 0.34 at the outlet of the deformation zone. This case models the drawing of load-bearing small-diameter pipes from large semifinished products. The result obtained here shows that to control the degree of collapse, it is necessary to use a pressure gradient.

The solutions presented in this article to the steady-state problem can easily be generalized to the case of examination of the process with allowance for equations expressing the energy and the weight of the fiber being formed.

NOTATION

z, Z, dimensional and dimensionless longitudinal coordinate; u_∞, U_∞, dimensional and dimensionless rate of extraction of the finished production; η, viscosity; η_0, minimum viscosity; μ, dimensionless distribution of yielding; h_0, thickness of the wall of the semifinished product; $\bar{r}_0$, mean radius of the semifinished product; u_0, feed of the semifinished product; σ, surface tension; ℓ, effective length of the heating zone; $\Delta p = p_1 - p_2$, pressure gradient between the channel and the environment; $Q = \sigma l / \eta_0 u_0 \sqrt{h_0 \bar{r}_0}$, dimensionless parameter characterizing the ratio of surface tension to viscous forces; $P = \Delta p l / 2\eta_0 u_0$, dimensionless pressure parameter; H, dimensionless wall thickness; R, dimensionless mean radius; U, dimensionless longitudinal velocity; K, capillarity coefficient of the semifinished product; $W = \ln U_\infty$, extraction parameter.

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